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THE COMPARATIVE INVESTIGATION FOR CALCULATING THE TEMPERATURES IN THE HOTTEST FUEL ROD OF BUSHEHR NUCLEAR POWER PLANT

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Abstract

In this paper, the effects of temperature distribution at the hottest fuel rod (Hot Fuel Pin) in the Bushehr nuclear power plant in Iran has been determined for defined thermal conductivities by using different mathematical functions. Whereas Hot Fuel Pin is one of the most important fuel rods from the heat temperature distribution aspect in the core of nuclear power plant, thus in this paper, the values of average temperature (T_{ave}) and temperatures of inner and outer diameters (T_{fi}, T_{fo}) for the Hot Fuel Pin, which is cylindrical shape, has been obtained truly. Two methods have been used for calculation of temperature, i.e., analytical and numerical methods. In the numerical method, computer

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programming by MATLAB software has been applied. Moreover, calculation of temperatures has been performed for two stages and results have been modified than previous stage and have been compared together. In every stage of calculations by two cited methods, the values of thermal conductivity (k) have been obtained by two methods that are: Lagrange and finite difference methods. One of the most important results is equality of k in the second stage of Lagrange method with the value of k in the first stage of finite difference method. The results show the obtained values of temperatures from analytical and numerical methods with defined k from Lagrange method are converged, but the mentioned values with defined k from finite difference method are diverged.

Nomenclature

T_{ave}: Average temperature of fuel rod.

T_{fi}: Temperature of fuel rod inside radius.

T_{fo}: Temperature of fuel rod outside radius.

r_i: Inside radius of fuel rod gap.

r_o: Outside radius of fuel rod.

$\dot{q}'$: Linear power.

$\dot{q}'''$: Power density.

k: Thermal conductivity coefficient.

α: Thermal diffusity $(\mathrm{m}^2/\mathrm{s})$.

Hottest fuel rod of reactor core: Hot Fuel Pin.

1. Introduction

This study is performed for a nuclear reactor VVER-1000 type, which is operating on steady state and the matter of its fuel rod is UO_2 (Benítez et al. [4]). Also, it is supposed $T_{ave} = 1800(\mathrm{K})$ (Final Safety Analysis Report [6]) and cited rod is: Hot Fuel Pin and the changes of temperature in tension of axial (z) and angle are cancelled.

There are following conditions:

$r_i = 0.75\text{mm},$

$r_o = 3.785\text{mm}.$

Height of fuel rod = 3.53m;

$\dot{q}' = 448\text{W/cm}$ (Final Safety Analysis Report [6]).

Boundary conditions (Safavisohi et al. [15]):

(1) If $r = r_i$, then $T = T_{fi}$, $\dfrac{\partial T(r, \varphi, z)}{\partial r} = 0.$

(2) If $r = r_o$, then $T = T_{fo}$.

In the following Figures 1 and 2, the lateral and front of fuel rod are shown:

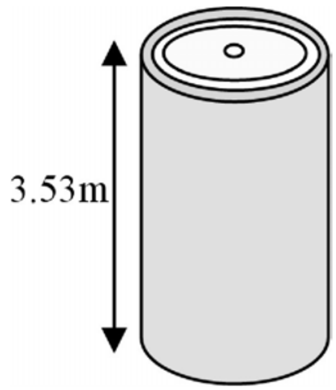

Figure 1. Fuel rod lateral surface.

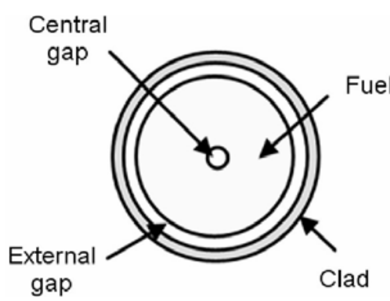

Figure 2. Fuel rod upper surface.

By application of above assumptions and initial assumption for T_{ave}, and by using the interpolation equations from two methods: Lagrange and finite difference, the value of k is calculated and by using it and by considering above initial conditions, the function of temperature distribution is defined (Sheng et al. [17]; Aguirre and Krause [3]; Langeroudi and Aghanajafi [8, 9]).

By application of temperature function and integrity method, value of T_{ave} is determined and compared with initial supposition. In the next stage, T_{ave}, which had been obtained from temperature distribution function, puts instead of supposed average temperature in the first, that is;

$$T_{ave} \approx \frac{T_{fi} + T_{fo}}{2} = 1800(\mathrm{K}), \tag{1}$$

and the mentioned procedure so is repeated and hereby the accurate value of T_{ave} is determined. Moreover, by determination of accurate value of T_{ave}, the values of T_{fi} and T_{fo} are determined. Also, inside and outside radii to very slight intervals are throughout divided and then values of T_{fi} and T_{fo} by numerical method are truly obtained and finally compared with obtained values from analytical method.

2. Materials and Methods

2.1. Method of solution

In cylindrical dimension, heat transfer equation is as follows:

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \varphi^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}'''}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \text{ (Wakil [20]).} \tag{2}$$

Once based on analytical method and the next time from the numerical method (Sadooghi and Aghanajafi [14]; Sharbati et al. [16]; Aghanajafi and Safavisohi [2]; Mehraban and Aghanajafi [10]; Abbassi and Aghanajafi [1]; Khaefinejad and Aghanajafi [7]) Equation (2) is solved and values of T_{ave}, T_{fo}, T_{fi}, and temperature distribution function are determined and compared together in the Hot Fuel Pin of Bushehr atomic power plant in Iran.

2.2. Calculations of temperature distribution by analytical method

By assumption of negligible temperature changes in axial tension (z) and also in angular tension (φ), one can write

$$\frac{\partial^2 T}{\partial z^2} = \frac{\partial^2 T}{\partial \varphi^2} = \frac{\partial T}{\partial t} \cong 0. \tag{3}$$

Thus,

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} = -\frac{\dot{q}'''}{k}. \tag{4}$$

So, by using the above condition, equation of T is produced

$$T = -\frac{\dot{q}''' r^2}{4k} + C_1 \ln r + C_2. \tag{5}$$

By using the mentioned boundary conditions in introduction section, values of C_1 and C_2 can be produced and then the Equation (5) as definite equation is determined. According to boundary condition (1)

$$C_1 = \frac{\dot{q}''' r^2}{2k}, \tag{6}$$

$$C_2 = T_{fi} + (2.164335 \times 10^{-6})\frac{\dot{q}'''}{k}. \tag{7}$$

For determination of numerical values of C_1 and C_2, values of $\dot{q}'''$, T_{fi}, and k are required. Whereas linear heat rate in Hot Fuel Pin was $\dot{q}' = 448\,\mathrm{W/cm}$ and heat flux for Hot Fuel Pin will be

$$\dot{q}''' = 103.6\mathrm{KW/m}^3. \tag{8}$$

Also, for calculation of T_{fi} according to conduction heat transfer, one can write

$$-kA\frac{\partial T}{\partial r} = \dot{q}'''V \text{ (Neil and Kazimi [12])}, \tag{9}$$

where A is lateral surface of fuel rod. According to boundary condition (2);

$$\ln r = r_o : T = T_{fo} \text{ so: } kA\frac{T_{fi} - T_{fo}}{r_o - r_i} = \dot{q}'''\pi(r_o^2 - r_i^2)L. \tag{10}$$

Thus by using this supposition, $T_{ave} \approx \frac{T_{fi} + T_{f0}}{2}$, one can write

$$k(2\pi r_o L)\frac{2(T_{fi} - T_{ave})}{r_o - r_i} = \dot{q}'''\pi(r_o^2 - r_i^2)L. \tag{11}$$

Now value of T_{ave} by using both temperature distribution function and related integrity (Formula (12)) is obtained and with T_{ave}, which had been supposed in first is compared, so

$$T_{ave} = \frac{1}{\text{v}}\iint T(r)d\text{v} \text{ (Olander [13])}, \tag{12}$$

$$T_{ave} = \frac{1}{\pi(r_o^2 - r_i^2)L}\int_{z_1=-\frac{L}{2}}^{Z_2=\frac{L}{2}}\int_{r_i}^{r_o} T(r)2\pi r dr dz. \tag{13}$$

Therefore, for obtaining k by interpolation with various temperatures, Table 1 is used.

Table 1. The changes of thermal conductivity coefficient of Hot Fuel Pin [2]

Temperature (K)	Thermal conductivity coefficient (k) in (W/m·K)
300	8.15
1100	3.75
1700	2.50
2700	2.65
3100	3.50

Supposed T_{ave} is 1800K as initial assumption. Also, in the further stages, new k through recent T_{ave} according to existing data of Table 1 is obtained.

2.3. Calculation of *k* by Lagrange method

The interpolation via Lagrange method for calculating the k by application either the information of Table 1 or Formulas (14) and (15) are performed

$$P_n(x) = \sum_{m=0}^{n} f(x_m) L_{n,m}(x), \quad m = 0, 1, \dots, n \text{ (Dusinberre [5])}, \quad (14)$$

$$L_{n,m}(x) = \frac{(x - x_0)(x - x_1)\cdots(x - x_{m-1})\cdots(x - x_{m+1})\cdots(x - x_n)}{(x_m - x_0)(x_m - x_1)\cdots(x_m - x_{m-1})\cdots(x_m - x_n)}. \quad (15)$$

Now, in this part, T_{ave_1} instead of conjectural value of T_{ave} is replaced and by considering T_{ave_1}, recent values of k_2, T_{ave_2}, T_{fi_2}, and T_{fo_2} are determined again according to carried out methods in the last stage.

2.4. Calculation of *k* by finite difference method

In this stage, value of k via finite difference method and by using both information of Table 1 and supposed T_{ave} is obtained. The interpolation via finite difference method based on Formula (16) is performed

$$P_{a,b,c,\dots,j}(x) = f_a + f_{a,b}(x - x_a) + f_{a,b,c}(x - x_a)(x - x_b)$$

$$+ \cdots + f_{a,b,c,\dots,j}(x - x_a)(x - x_b)(x - x_c)\cdots(x - x_{j-1})$$

$$f_{a,b,c,\dots,j} = \frac{f_{b,c,\dots,j} - f_{a,b,c,\dots,(j-1)}}{x_j - x_a} \text{ (Dusinberre [5]).} \quad (16)$$

So by having T'_{ave_1}, the values of k'_2 and T'_{ave_2}, T'_{fi_2}, and T'_{fo_2} according to performed methods in the last stage are determined.

2.5. Calculation of temperature distribution by numerical method

In this stage, by using the numerical solving for $\frac{\partial T}{\partial r}$, $\frac{\partial^2 T}{\partial r^2}$ can write

$$\frac{\partial T}{\partial r} = \frac{T_{n+1} - T_{n-1}}{2\Delta r}, \tag{17}$$

and

$$\frac{\partial^2 T}{\partial r^2} = \frac{T_{n+1} - 2T_n + T_{n-1}}{(\Delta r)^2} \text{ (Mitchell and Griffiths [11]; Smith [19]).} \tag{18}$$

Thus Equation (2) changes to this form

$$\left(\frac{T_{n+1} - 2T_n + T_{n-1}}{(\Delta r)^2}\right) + \frac{1}{r}\left(\frac{T_{n+1} - T_{n-1}}{2(\Delta r)}\right) + \frac{\dot{q}'''}{k} = 0, \tag{19}$$

where

$$\Delta r = r_{n+1} - r_n. \tag{20}$$

The between distance of inside and outside diameters of fuel rod to small intervals is divided, that is, $\Delta r = 10^{-5}$ m and via computer programming by MATLAB software, the values of temperature are produced in the following points:

$$r = r_i = 75\Delta r \text{ and } r = r_o = 3785\Delta r \text{ (Shoichiro [18]).}$$

Therefore, the values of T_{fi}, T_{fo}, and T_{ave} for numerical method, the same of analytical method for two stages are determinable.

3. Result and Discussion

With comparison of all the produced parameters for Hot Fuel Pin, Tables 2-5 are produced as following:

Table 2. The obtained results in the first stage of computation by numerical and analytical methods

The first stage of calculation by numerical method (defined k by finite difference method) $k_1' = 2.475(\mathrm{W/mK})$	The first stage of calculation by analytical method (defined k by Lagrange method) $k_1 = 2.564(\mathrm{W/mK})$	Percent of error
$T_{ave_1} = 1822.5(\mathrm{K})$	$T_{ave_1} = 1880.0(\mathrm{K})$	3.05%
$T_{fi_1} = 2048.6(\mathrm{K})$	$T_{fi_1} = 2023.7(\mathrm{K})$	– 1.23%
$T_{fo_1} = 1708.3(\mathrm{K})$	$T_{fo_1} = 1736.3(\mathrm{K})$	1.61%
$T_1 = -10101.4r^2 + 20202.8r^2 \ln r + 1952$		

Table 3. The obtained results in the first stage of computation by numerical and analytical methods

The first stage of calculation by numerical method (defined k by Lagrange method) $k_1 = 2.564(\mathrm{W/mK})$	The first stage of calculation by analytical method (defined k by finite difference method) $k_1' = 2.475(\mathrm{W/mK})$	Percent of error
$T'_{ave_1} = 1802.2(\mathrm{K})$	$T'_{ave_1} = 1912.2(\mathrm{K})$	5.75%
$T'_{fi_1} = 2023.7(\mathrm{K})$	$T'_{fi_1} = 2098.0(\mathrm{K})$	3.54%
$T'_{fo_1} = 1701.0(\mathrm{K})$	$T'_{fo_1} = 1726.4(\mathrm{K})$	1.47%
$T_1' = -10464.6r^2 + 21082.6r^2 \ln r + 2098$		

Table 4. The obtained results in the second stage of computation by numerical and analytical methods

The second stage of calculation by numerical method (defined k by finite difference method) $k'_2 = 2.489(\mathrm{W/mK})$	The second stage of calculation by analytical method (defined k by Lagrange method) $k_2 = 2.475(\mathrm{W/mK})$	Percent of error
$T_{ave_2} = 1807.4(\mathrm{K})$	$T_{ave_2} = 1912.2(\mathrm{K})$	5.48%
$T_{fi_2} = 2036.0(\mathrm{K})$	$T_{fi_2} = 2098.0(\mathrm{K})$	2.95%
$T_{fo_2} = 1703.0(\mathrm{K})$	$T_{fo_2} = 1726.4(\mathrm{K})$	1.35%
	$T_2 = -10464.6r^2 + 21082.6r^2 \ln r + 2098$	

Table 5. The obtained results in the second stage of computation by numerical and analytical methods

The second stage of calculation by numerical method (defined k by Lagrange method) $k_2 = 2.475(\mathrm{W/mK})$	The second stage of calculation by analytical method (defined k by finite difference method) $k'_2 = 2.489(\mathrm{W/mK})$	Percent of error
$T'_{ave_2} = 1822.5(\mathrm{K})$	$T'_{ave_2} = 1898.8(\mathrm{K})$	4.01%
$T'_{fi_2} = 2048.6(\mathrm{K})$	$T'_{fi_2} = 2036.4(\mathrm{K})$	− 0.59%
$T'_{fo_2} = 1708.3(\mathrm{K})$	$T'_{fo_2} = 1761.2(\mathrm{K})$	3.00%
	$T'_2 = -10405.8r^2 + 20811r^2 \ln r + 2036.4$	

Tables 2-5 show results by two methods: Lagrange and finite difference. The values of T_{ave}, T_{fi}, and T_{fo} in each stage of computation are converged to the last stage and are modified in every stage than previous stage. One of the most important results is equality of obtained k in the second stage of Lagrange method with the value of obtained k in the first stage of finite difference method, that means $k'_1 = k_2$.

In Figures 3-6, the related graphs for comparison of obtained temperatures by analytical and numerical methods with defined k from Lagrange and finite difference methods in the every stage are shown. The Figures 3-6 are following:

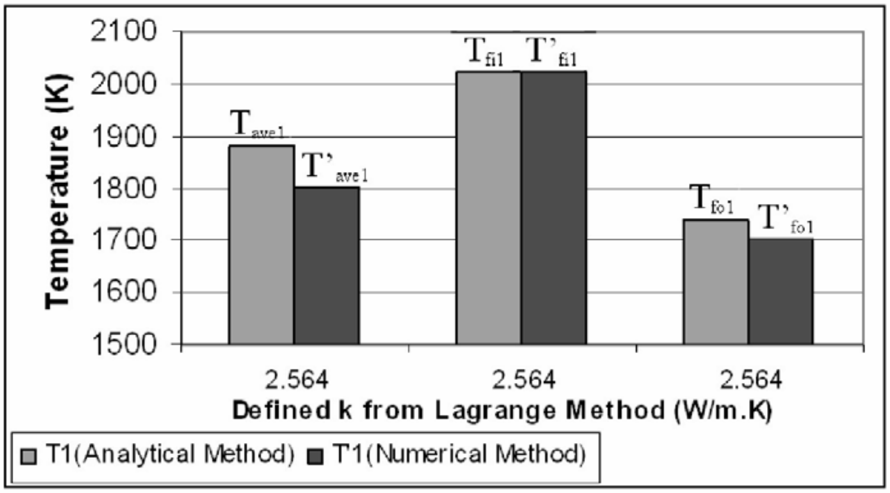

Figure 3. The obtained temperatures by analytical and numerical methods in the first stage with defined k from Lagrange method.

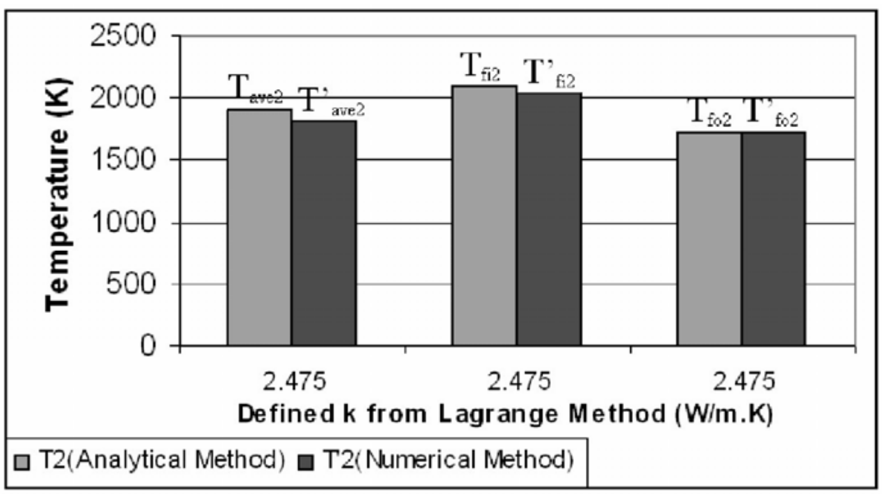

Figure 4. The obtained temperatures by analytical and numerical methods in the second stage with defined k from Lagrange method.

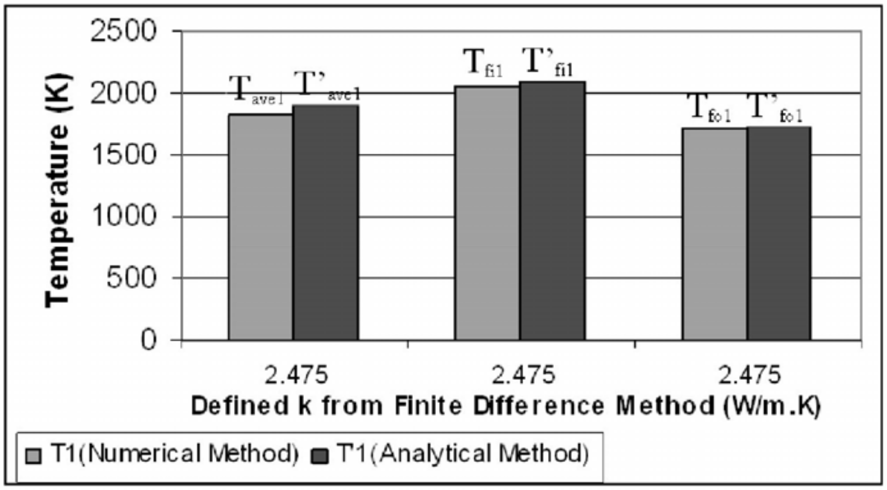

Figure 5. The obtained temperatures by numerical and analytical methods in the first stage with defined *k* with finite difference method.

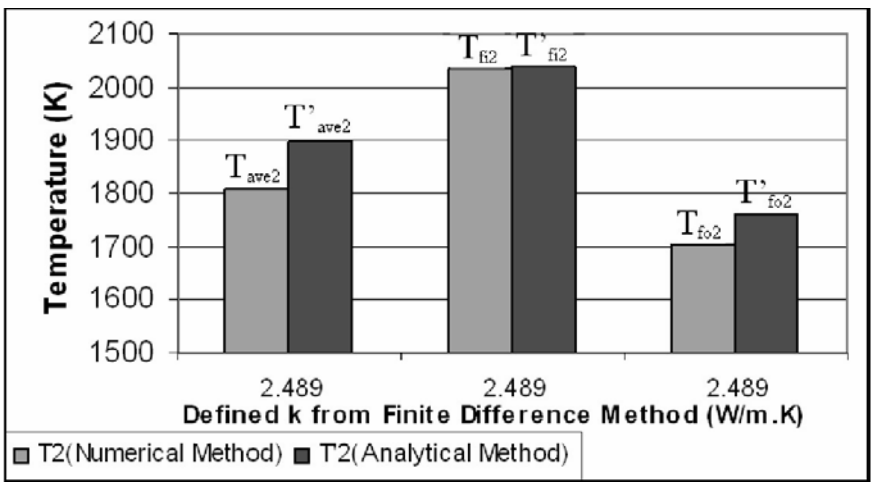

Figure 6. The obtained temperatures by numerical and analytical methods in the second stage with defined *k* from finite difference method.

In Figures 7-10, the curves of temperatures equations, namely; T_1, T_1', T_2, and T_2' are shown:

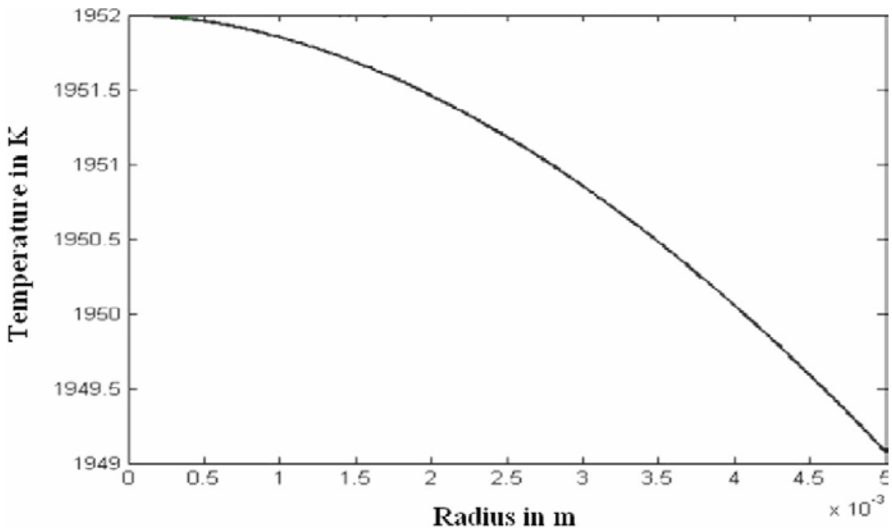

Figure 7. The curve of equation T_1.

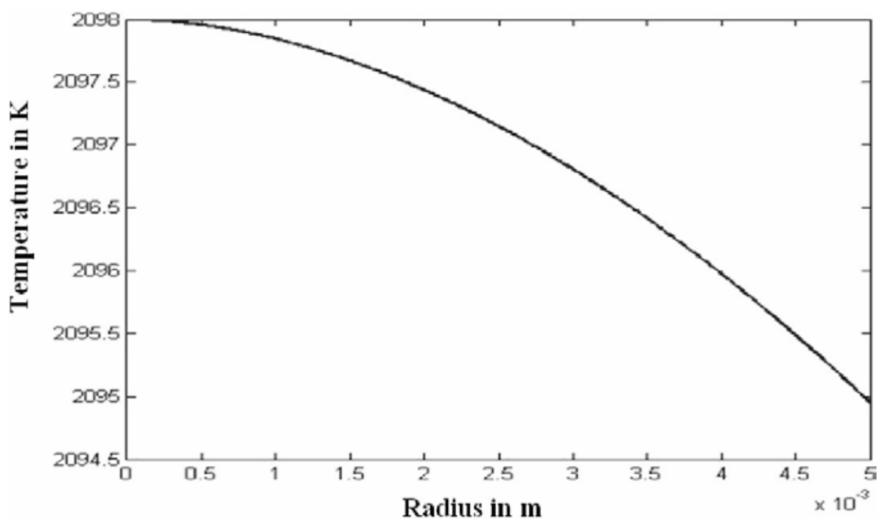

Figure 8. The overlapped curves of both equations T_1' and T_2.

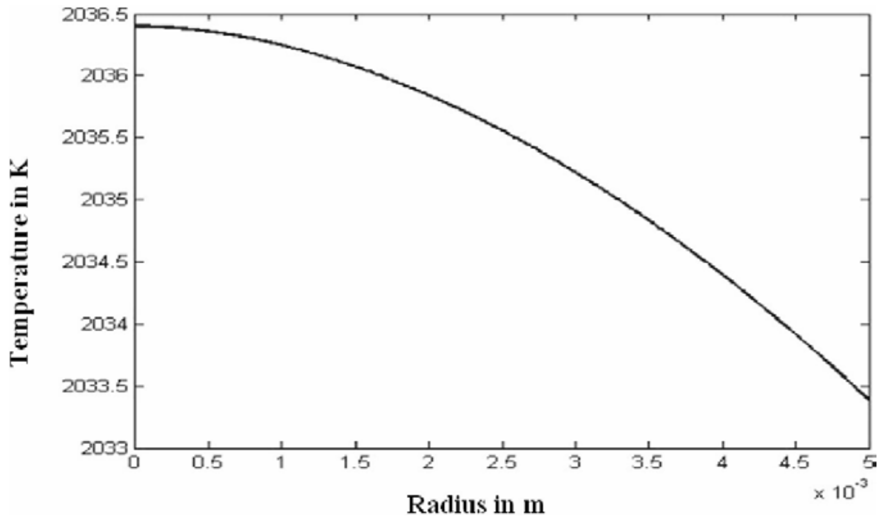

Figure 9. The curve of equation T_2'.

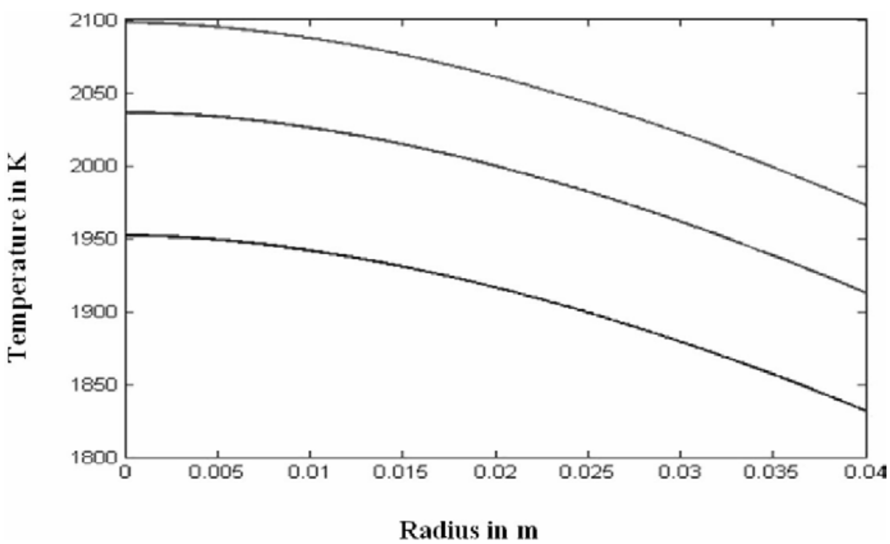

Figure 10. The curves of four equations T_1, T_1', T_2, and T_2'.

4. Conclusion

The results show the values of temperatures $(T_{ave}, T_{fi}, \text{ and } T_{fo})$ in the Hot Fuel Pin truly. The results show that by application, the defined k from Lagrange method in the first and second stages of computation, the obtained values of temperatures from analytical and numerical methods are converged together. But by application, the defined k from finite difference method in both stages the obtained values of temperatures from analytical and numerical methods than the previous stage are diverged. Also, it is observed that both curves T_1' and T_2 are overlapped. Therefore, one can say the appropriate determined values for T_{ave}, T_{fi}, and T_{fo} of Hot Fuel Pin are values in which the second stage of computation with defined k from Lagrange method and in which the first stage with defined k from finite difference method, that means; $T'_{ave_1} = T_{ave_2} = 1912.2(\text{K})$, $T'_{fi_1} = T_{fi_2} = 2098.0(\text{K})$, and $T'_{fo_1} = T_{fo_2} = 1726.4(\text{K})$.

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References

[1] H. Abbassi and C. Aghanajafi, Evaluation of heat transfer augmentation in a nanofluid-cooled microchannel heat sink, Journal of Fusion Energy 25 (2006), 187-196.

[2] C. Aghanajafi and B. Safavisohi, Optimization of production process of a semitransparent layer containing foam by considering radiation and using finite difference method, Journal of Fusion Energy 25 (2006), 213-217.

[3] M. Aguirre and J. Krause, General transformation theory of Lagrangian mechanics and the Lagrange group, International Journal of Theoretical Physics 30 (1991), 495-509.

[4] J. Benítez, P. Martínez and H. Pérez, International cooperation on control for safe operation of nuclear research reactors, Journal of Progress in Nuclear Energy 46 (2005), 321-327.

[5] G. M. Dusinberre, Heat Transfer Calculation by Finite Difference, International Textbook, (1961), 58-70.

[6] FSAR (Final Safety Analysis Report), Chapter 4, 2003.

[7] B. Khaefinejad and C. Aghanajafi, Laminar flow with simultaneously effects of mixed convection and thermal radiation in a channel, Journal of Fusion Energy 28 (2009), 83-90.

[8] H. Langeroudi and C. Aghanajafi, Analysis and optimization laminar viscous flow through a channel, Journal of Fusion Energy 25 (2006), 155-164.

[9] H. Langeroudi and C. Aghanajafi, Thermodynamic's second law analysis for laminar non-Newtonian fluid flow, Journal of Fusion Energy 25 (2006), 165-173.

[10] P. Mehraban and C. Aghanajafi, The effect of thermal radiation on nanofluid cooled microchannels, Journal of Fusion Energy 28 (2009), 91-100.

[11] A. R. Mitchell and D. F. Griffiths, The Finite Difference Methods in Partial Differential Equations, Wiley- Inter Science, (1980), 135-137.

[12] E. T. Neil and M. Kazimi, Nuclear Systems 1 Thermal Hydraulic Fundamentals, Hemisphere Publishing Corporation, (1990), 295-337.

[13] R. Olander, Fundamental Aspects of Nuclear Reactor Fuel Elements, (1976), 80-82.

[14] P. Sadooghi and C. Aghanajafi, Coating effects on transient cooling a hot spherical body, Journal of Fusion Energy 22 (2003), 59-65.

[15] B. Safavisohi, E. Sharbati, C. Aghanajafi and R. Khatami, Effects of boundary conditions on thermal response of a cellulose acetate layer using Hottel's zonal method, Journal of Fusion Energy 25 (2006), 145-153.

[16] E. Sharbati, B. Safavisohi and C. Aghanajafi, Transient heat transfer analysis of a layer by considering the effect of radiation, Journal of Fusion Energy 23 (2004), 207-215.

[17] L. Sheng, R. Sheng and S. Song, Reduced projection augmented Lagrange bi-conjugate gradient method for contact and impact problems, Journal of Applied Mathematics and Mechanics 30 (2007), 1101-1108.

[18] N. Shoichiro, Applied Numerical Methods with Software, Prentice-Hall International, Inc. 70 (1991).

[19] D. G. Smith, Numerical Solution of Partial Differential Equations, Oxford University Press, (1978), 40-48.

[20] E. Wakil, Nuclear Heat Transfer, Hemisphere Publishing Corporation, (1991), 103-129.

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